Coupled Radial Basis Functions and Cover Scheme Methods Applied to Solving Optimal Control Problems

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Abstract

- A Coupled Radial Basis Function (CRBF) and cover scheme method is presented for solving optimal control problems in astrodynamics applications
- Aims to overcome inherent challenges with current solving methods – to include requiring *a priori* information, good initial guess, nodal distribution dependency, sensitivity to shape parameter, etc
- Method applied to solving HCW Equations, Optimal Attitude Control Problem, and Low Thrust Orbit Transfer Problem

Optimal Control

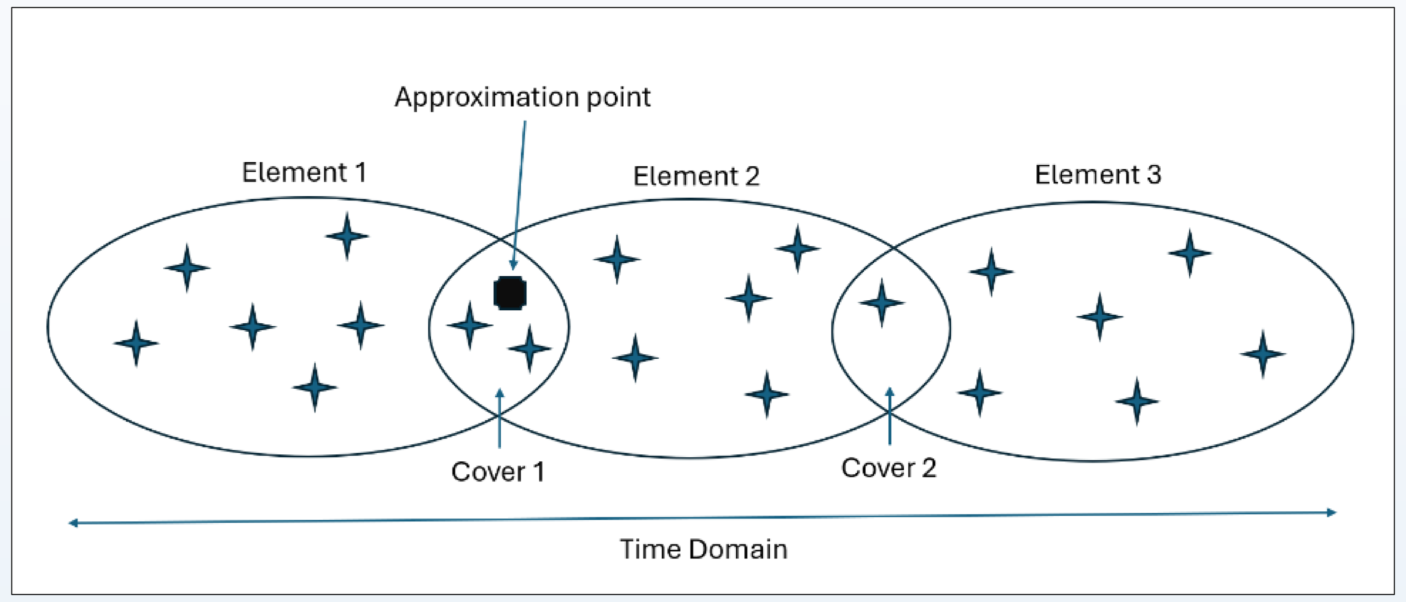
- Seeks to find the control that minimizes (or maximizes) performance criterion while satisfying constraints
- Applied in solving more complex dynamic systems, such as nonlinear multiple-input multiple-output (MIMO) systems
- Due to complexity, numerical methods are often employed
  - Nonlinear Programming Problem
  - Two-point boundary value problem (TPBVP)
- Examples: minimal fuel consumption, flight optimization, orbit transfer
- Numerical methods: indirect methods and direct methods
- Collocation: Local collocation and global collocation
- Current methods: dependent on nodal distribution, initial guess, shape parameter, etc

Coupled Radial Basis Functions

- To solve optimal control problems via CRBF-collocation methods:
  - Formulate optimal control problem
  - Obtain necessary conditions for optimality
  - Transcribe resulting TPBVP into system of nonlinear algebraic equations
    - Using CRBFs
    - Approximate states, co-states, control, and boundary conditions
- Solve using standard solver/numerical methods

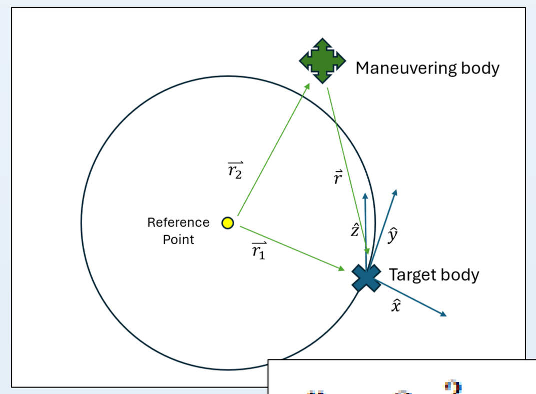
Cover Scheme Method

- Problem domain divided into segments called "elements" with overlapping regions called "covers"
- Locally collocated
- Problem domain is the union of all covers
- Intersections can include multiple nodes
  - Enforces continuity at boundaries
- # nodes, covers, intersections can be varied to reduce approximation error

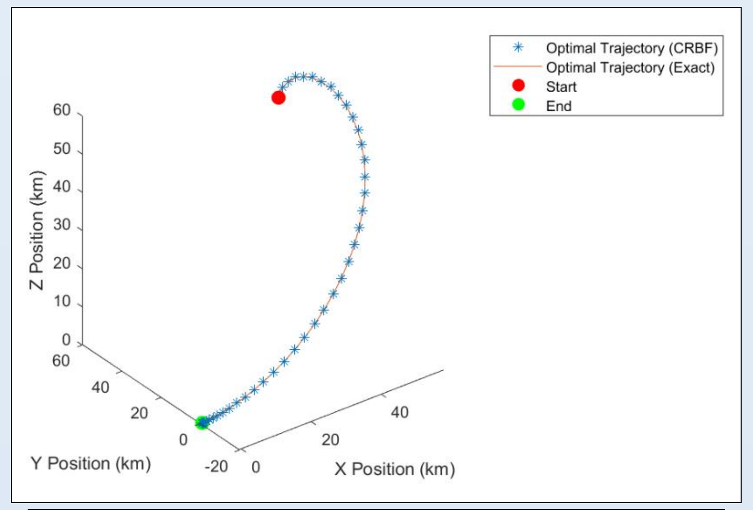


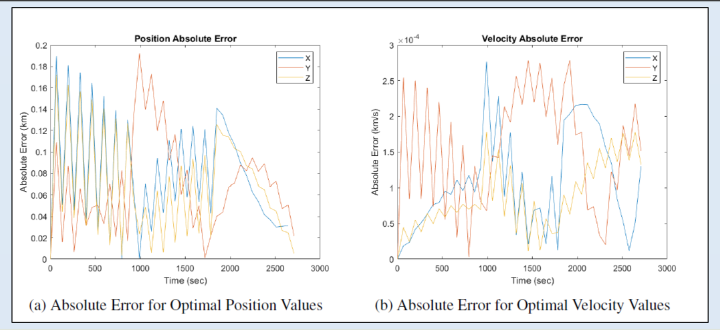
Results

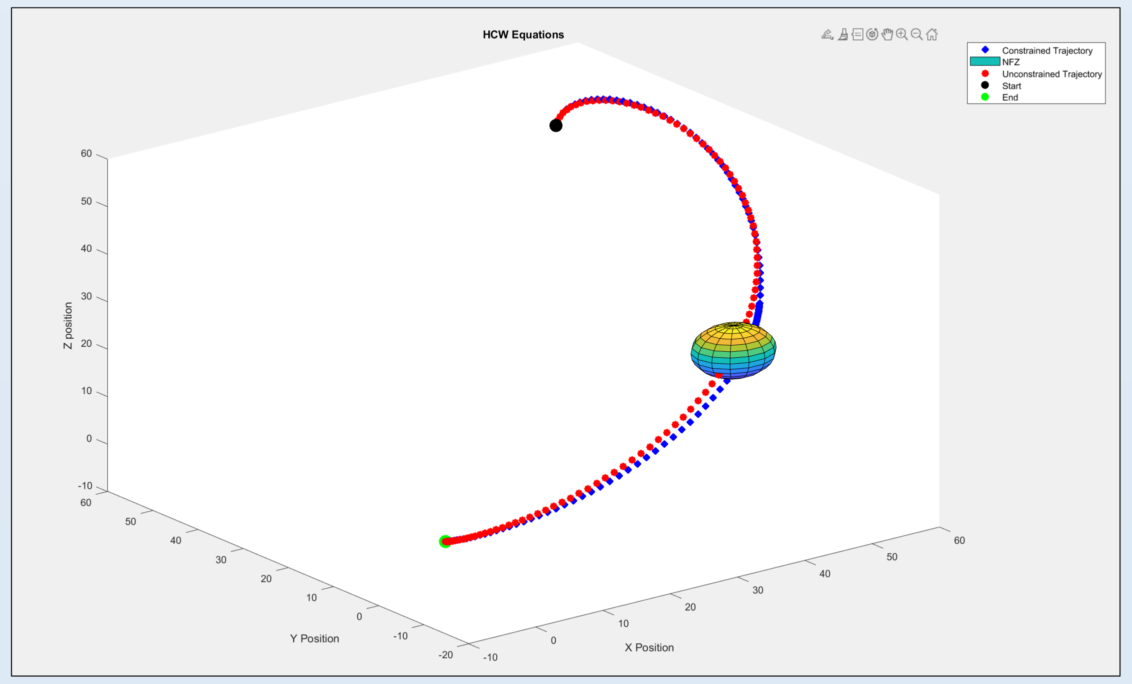
- **Hill-Clohessy-Wiltshire (HCW) Equations**
- Linearized equations for relative orbital motion
- Unconstrained problem- for rendezvous and docking case
- Constrained problem – no-fly-zone avoidance
- Insensitive to initial guess + nodal distribution
- Absolute error on magnitude of 10^{-4} compared to closed-form solution



$$\ddot{x} = 3n^2x + 2n\dot{y}$$
$$\ddot{y} = -2n\dot{x}$$
$$\ddot{z} = -n^2z$$





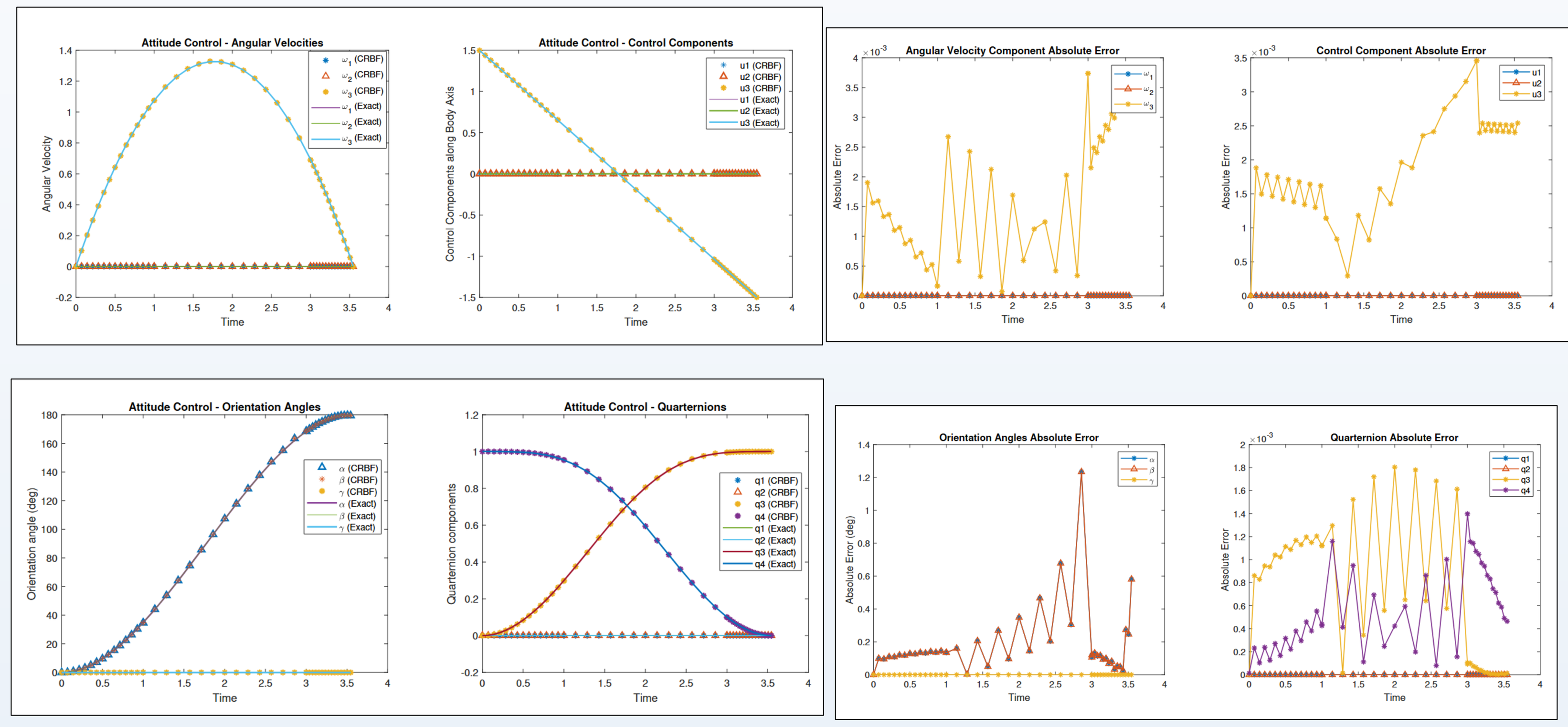


Results (continued)

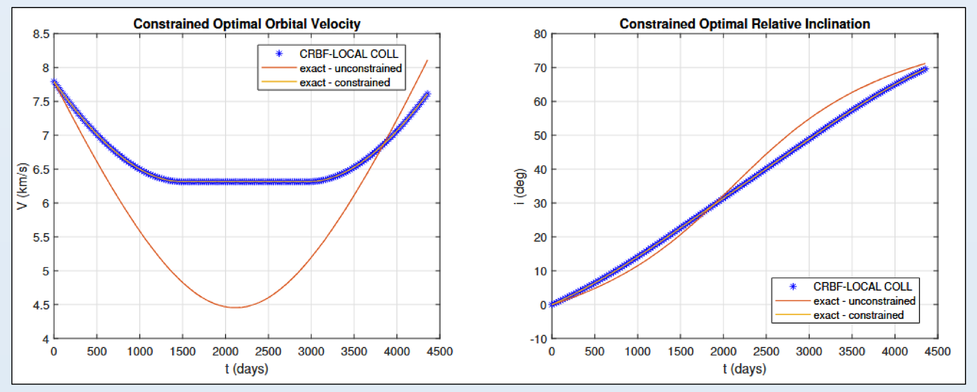
- **Optimal Attitude Control**
- Kinematics and dynamics to achieve reorientation of rigid body + minimize control effort
- Error on magnitude of 10^{-3} compared to closed-form solution

$$\dot{q}_1 = \frac{1}{2}(\omega_1 q_4 - \omega_2 q_3 + \omega_3 q_2)$$
$$\dot{q}_2 = \frac{1}{2}(\omega_1 q_3 - \omega_2 q_4 + \omega_3 q_1)$$
$$\dot{q}_3 = \frac{1}{2}(-\omega_1 q_2 - \omega_2 q_1 + \omega_3 q_4)$$
$$\dot{q}_4 = \frac{1}{2}(-\omega_1 q_1 - \omega_2 q_2 - \omega_3 q_3)$$

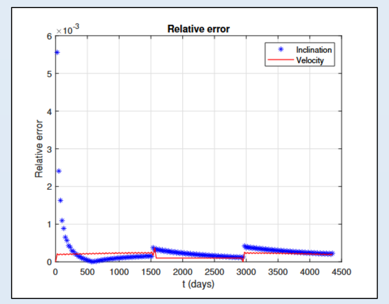
$$\tau_1 = I_1\dot{\Omega}_1 - (I_2 - I_3)\Omega_2\Omega_3$$
$$\tau_2 = I_2\dot{\Omega}_2 - (I_3 - I_1)\Omega_3\Omega_1$$
$$\tau_3 = I_3\dot{\Omega}_3 - (I_1 - I_2)\Omega_1\Omega_2$$



- **Low Thrust Orbit Transfer**
- Circular low-thrust orbit transfer with altitude constraint implemented for intermediate orbits
- Solution shows accuracy and robustness
- Error on magnitude of 10^{-3} compared to closed-form solution



$$\dot{a} = \frac{2afcos(\beta)}{V}$$
$$\dot{i} = \frac{cos(\theta)fsin(\beta)}{V}$$
$$\dot{\theta} = n$$



Future Work

- Investigate the effects of shared nodes in cover regions
- Explore ways to make the cover scheme method adaptive
- Create an error estimate model