A STUDY OF CISLUNAR PERIODIC ORBITS IN THE CIRCULAR RESTRICTED THREE-BODY PROBLEM

Nicole Weeden Martins
University of Central Florida, Orlando, FL

Introduction

- The **Circular Restricted Three Body Problem (CR3BP)** is a simplified version of the N-Body Problem, by adding constraints [1].
- This model only has three bodies; one of the bodies is considered massless if compared to the others, and the two largest bodies - primaries - rotate in a circular orbit around the center of mass - barycenter - of the system. In this poster, the system consists of Earth, the Moon, and a spacecraft, referred to as cislunar space.
- The only unknowns of this model are **related to the spacecraft's path** since the motion of the primaries is known by the two-body problem solution [2].
- The Lagrangian points are derived from the Equations of Motion (EOMs) by setting the velocities, accelerations, and z component of the position equal to zero, giving five different solutions/points [2][5][6].
- The **Jacobi constant is the only constant of motion** for this model, serving as a powerful tool to understand and verify the behavior of a spacecraft, acting like the "DNA" of the orbit [2][3][4].
- This model does not account for all the existing perturbations, which means that if applied in reality, the spacecraft would **need slight adjustments** to maintain its path.
- The periodic orbits in the CR3BP are grouped based on their shapes, locations, and stabilities. These groups are referred as families.
- The Gateway mission is to be positioned on the **Near Rectilinear Halo Orbits (NRHOs)**, a subset of the Halo Orbits family.
- Other key families that could be relevant to the Artemis program are the **Distant Retrograde Orbits (DROs)** and the **Butterfly Orbits**, as they could be used as relatively cheaper transfer orbit options [8][9].

Methods

- The Equations of Motion (EOMs) derived from the CR3BP model:

$$\ddot{x} = 2\dot{y} + x - \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x-(1-\mu))}{r_2^3}, \quad \ddot{y} = -2\dot{x} + y - \frac{(1-\mu)y}{r_1^3} - \frac{\mu y}{r_2^3}, \quad \ddot{z} = -\frac{(1-\mu)z}{r_1^3} - \frac{\mu z}{r_2^3}$$

- The EOMs are in the **synodic frame**, which is a **non-dimensional frame** that has its center at the barycenter of the system, and the location of the primaries is constant in time. The mass ratio of this system is $\mu = 0.01215052404$, and the coordinates of Earth and Moon are $[-\mu, 0, 0]$, $[1-\mu, 0, 0]$, respectively. The time unit (TE) is 382981 seconds, found by dividing the Period of the Moon by 2π.
- A code that solves for the EOMs given an initial condition for a periodic orbit, Position (x, y, z) and Velocity (x, y, z) was created.
- The code propagates the orbit finds its period, Jacobi constant, and plot those. For the numerical integration ODE45 was used, and for the other calculations multiple functions were created.
- The Jacobi constant value for all the orbits fluctuated in around 10^{-6} decimals, so they were averaged before adding it to each plot.
- The initial conditions, the Lagrangian coordinates, and μ were taken **from the Jet Propulsion Laboratory (JPL)** Three-Body Periodic orbits database [10].
- This database was used to **validate the results**, Jacobi constant and the Period, and to evaluate how close the initial and final conditions were.
- The orbits are visualized in a 3D plot and depending on their locations, the primaries or the Lagrangian points were added to ensure a better result interpretation.

Results

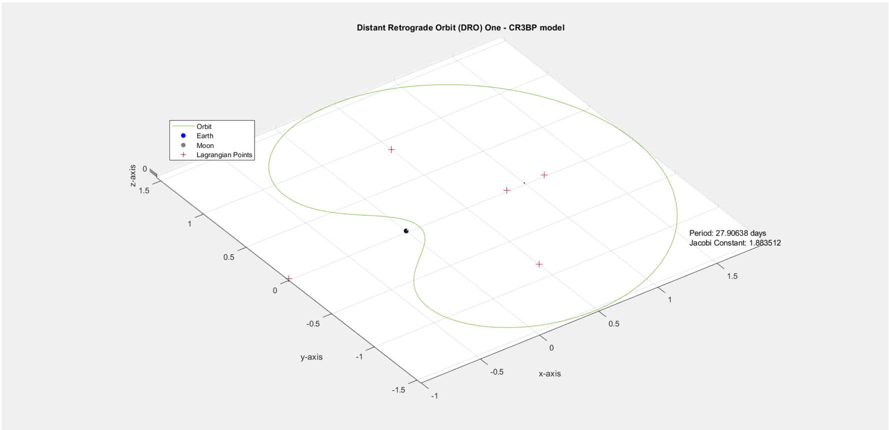

Figure 1: Distant Retrograde Orbit (DRO) 313 from JPL's website

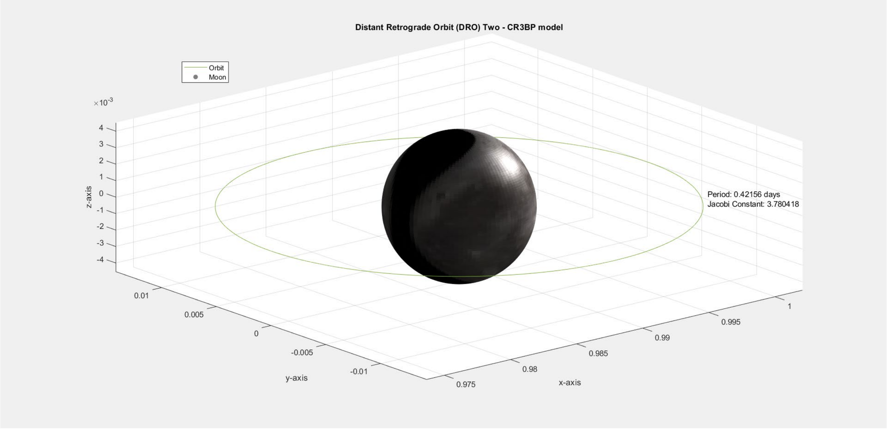

Figure 2: Distant Retrograde Orbit (DRO) 1020 from JPL's website

Table 1: Results obtained using the MATLAB code after one revolution, and the Period, Position, and Velocity are in non-dimensional units.

Code	DRO One	DRO two	Butterfly One	Butterfly Two	NRHO One	NRHO Two
Period	6.295642E+00	9.510400E-02	4.849971E+00	8.831549E+00	1.900588E+00	2.106034E+00
Jacobi Cons	1.883512E+00	3.780418E+00	3.063873E+00	2.928535E+00	2.983627E+00	2.950908E+00
Final position X	9.165727E-02	9.736433E-01	9.263977E-01	1.038228E+00	9.331088E-01	9.307215E-01
Final position Y	-4.817030E-05	-1.428232E-05	2.574174E-04	6.963240E-04	-2.064545E-04	-1.877343E-04
Final position Z	2.381906E-32	-2.699959E-33	1.489778E-01	2.370133E-01	2.433557E-01	2.827207E-01
Final velocity X	9.684073E-04	-9.433273E-04	4.738666E-04	9.996636E-04	-3.502231E-04	-3.366634E-04
Final velocity Y	4.145391E+00	9.393308E-01	-1.554421E-01	-2.906538E-01	9.272659E-02	8.186583E-02
Final velocity Z	-2.082754E-31	2.983747E-32	9.996633E-04	9.345702E-04	9.997387E-04	9.997531E-04

Table 2: Data collected from the JPL website for each specific orbit plotted, the Period, Position, and Velocity are in non-dimensional units.

JPL data	DRO One	DRO two	Butterfly One	Butterfly Two	NRHO One	NRHO Two
Period	6.295652E+00	9.511821E-02	4.851625E+00	8.833945E+00	1.902813E+00	2.108326E+00
Jacobi Cons	1.883512E+00	3.780418E+00	3.063873E+00	2.928535E+00	2.983627E+00	2.950908E+00
Initial position X	9.165728E-02	9.736433E-01	9.263983E-01	1.038229E+00	9.331084E-01	9.307211E-01
Initial position Y	-4.253164E-27	-1.413784E-28	-2.268074E-28	-4.138555E-28	1.468078E-27	-4.008754E-25
Initial position Z	2.375627E-32	-2.643816E-33	1.489785E-01	2.370145E-01	2.433568E-01	2.827218E-01
Initial velocity X	-2.777072E-12	4.654727E-13	-8.643183E-16	-4.334901E-14	-1.530801E-12	-1.608188E-10
Initial velocity Y	4.145391E+00	9.393313E-01	-1.554439E-01	-2.906567E-01	9.272758E-02	8.186675E-02
Initial velocity Z	-1.851571E-31	4.619839E-32	-3.158578E-14	-9.625826E-14	8.198274E-12	7.803142E-10

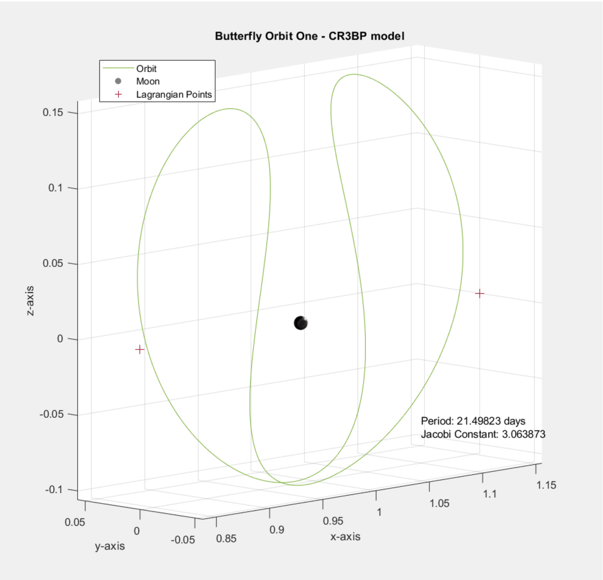

Figure 3: Butterfly Orbit 717 from JPL's website

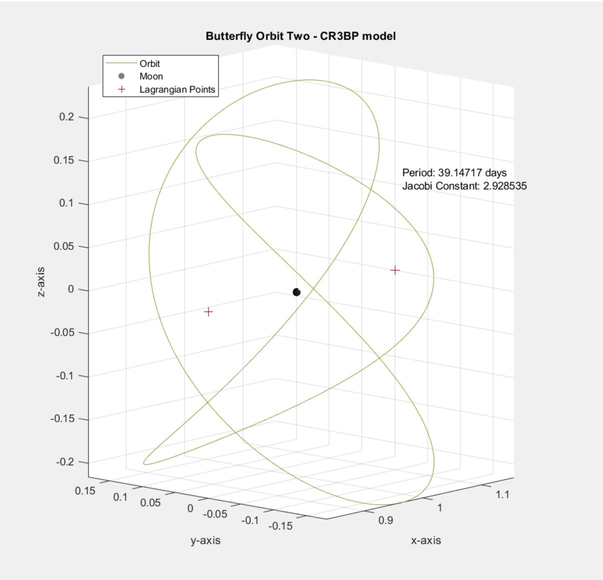

Figure 4: Butterfly Orbit 219 from JPL's website

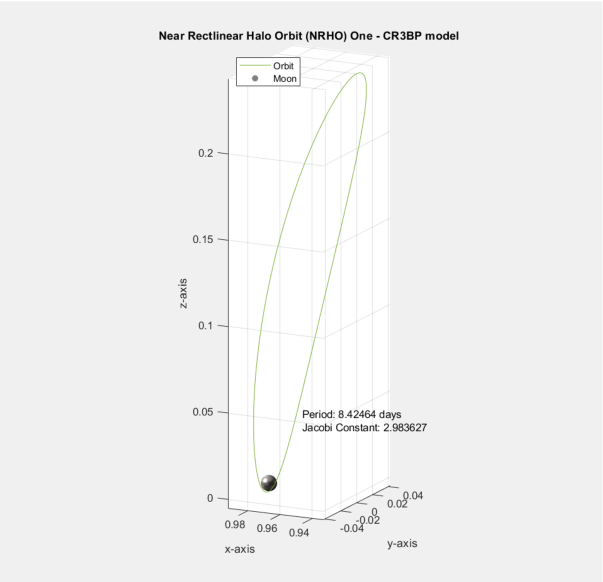

Figure 5: A Near Rectilinear Halo Orbit (NRHO) 873 from JPL's website

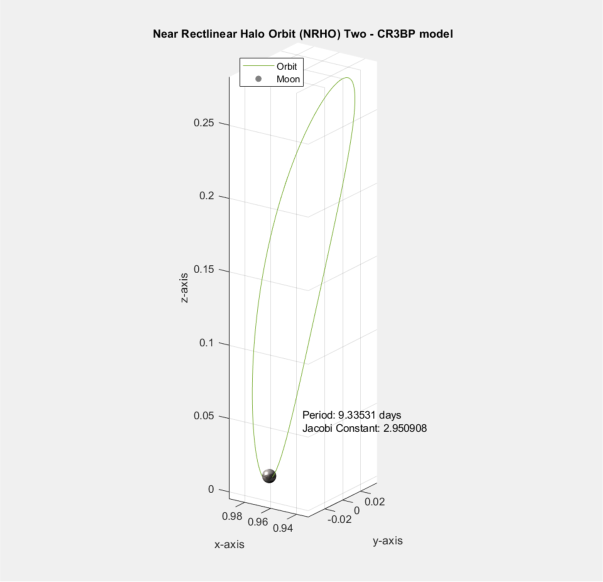

Figure 6: A Near Rectilinear Halo Orbit (NRHO) 856 from JPL's website

Discussion

- Both DROs chosen have similar geometries - circular shapes - but have **periods that differ quite a bit**; the orbit in Figure 1 has a period of around 27.91 days, while the Period for Figure 2 is around 0.42 days (10.12 hours).
- Furthermore, Figure 1 has a wide geometry, passing very far from the Moon and arguably close to Earth. Meanwhile, the other DRO stays around the premises of the Moon.
- Analyzing the Butterfly family, it is possible to see a slight change in the geometry between the two orbits plotted, which is not seen in the DROs. **Both present curves in their path**, suggesting the name of the family; However, Figure 4 has many more turns than Figure 3, which presents only two wing-shaped paths.
- The Periods of Figure 3 and 4 21.5 and 39.15 days, respectively, may seem quite large; however, it is essential to **recall the magnitude of the trajectories** as two Lagrangian points, L_1 and L_2, surround their paths.
- The two orbits of the Halo Orbits family, NRHO subgroup, were quite similar in shape, location, and period but with a **key difference**: Figure 6 has a path that goes through the surface of the Moon - not feasible in a real mission - while Figure 5's trajectory stays relatively close to the surface but does not go through.
- The numerical integration alone gives insights if the orbit passes near or through a primary as it **demands a much smaller time step** to function and adequately plot the orbit.
- A comparison error was calculated using the JPL data (Table 2) to check the accuracy of the results (Table 1).
- The Period and Jacobi constant values obtained from the code were **extremely close** to the actual values for all the orbits plotted, with the errors ranging from 0.00016% to 0.11695% for the Period and $9.8024 * 10^{-8}\%$ to $1.85953 * 10^{-6}\%$ for the Jacobi constant.
- The error percentages for the position and velocity vectors rose in the value for position y, velocity x, and velocity z for all the orbits plotted, causing the error percentages to be relatively high. If the raw numbers are analyzed, the increase is not as significant since both could be **rounded to zero**.

Conclusion & Future Works

- This study shows how even orbits from the **same family have different characteristics** and could serve for various types of space missions.
- The gateway mission requires a periodic orbit that is easily accessible – from both Earth and Moon - stable and has good communication with Earth; the NRHO subgroup portrays these characteristics, as **shown in Figure 5**.
- These results confirm how important a close analysis of a family of orbits is before any trajectory is chosen.
- .Future work studies are planned to further this analysis by using the **Elliptic Restricted Three Body Problem (ER3BP).**

Acknowledgements & References

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References